




Doubly Sparse (DS-Softmax): Sparse Mixture of Sparse Experts for Efficient Softmax Inference

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Softmax Inference Problem

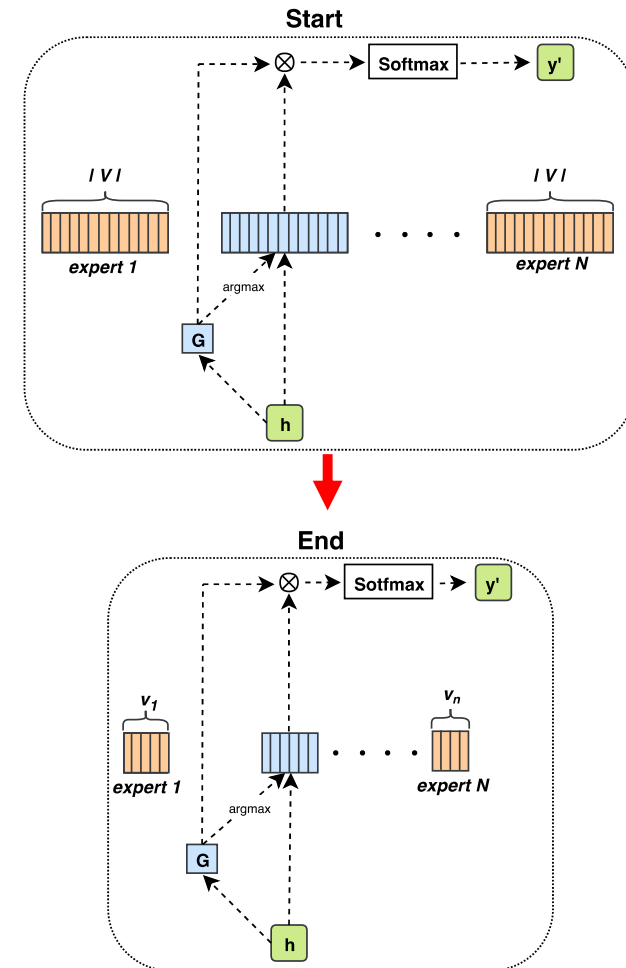
- **Softmax Inference:** $\operatorname{argmax}_c \frac{\exp(W_c h)}{z}$, where $z = \sum_i^N \exp(W_i h)$
- **Linear Complexity:** $O(N)$, depends on number of output classes
- Softmax as computational **Bottleneck** example:
 - Dataset: Wiki-2, Number of Words = 33k
 - Model: Two layers RNN, hidden size = 200
 - Softmax Computation counts more than **98%**
- **Common** in Real Applications:    ...
- **Traditional solutions**
 - Treat it as Maximum Inner Product Search (MIPS) in learned Softmax
 - Drawback: they suffer the **accuracy-speedup trade-off**
 - Example: Fast Graph Decoder¹ achieves only **~ 2x** in high accuracy

Doubly Sparse (DS-) Softmax

DS-Softmax: A **learning-based** model which adapts Softmax embedding into **hierarchical** structure for a better trade-off.

Implementation: A mixture of expert model where only the expert with **highest** mixture/gating value is activated

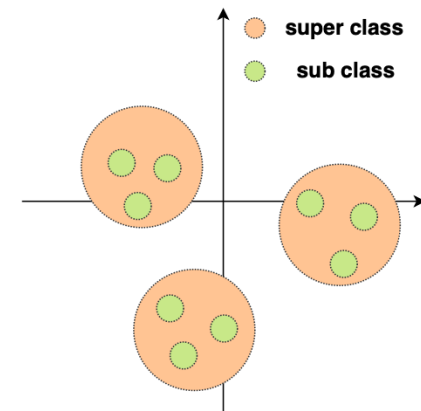
- **Initialization:** each expert contains full output space
- **Training:** iteratively pruning that each expert finally contains a subset of output classes. Then fast search can be achieved



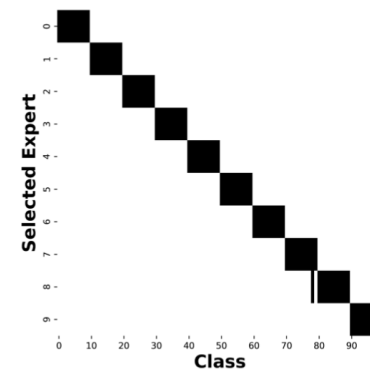
Result – Synthetic Dataset

Dataset: two-level hierarchy

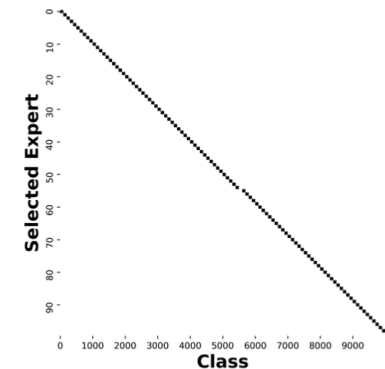
- **Generation:**
 - Sample **super classes**
 - Sample **sub** around **super**
 - Sample training points
- Super class label is **hidden**
- **Two sizes:** 100 classes (10 x 10)
and 10, 000 (100 x 100)
- **DS-Softmax** can fully capture the synthetic hierarchy



(a) Synthetic Data Generation



(b) Results on 10 x 10



(c) Results on 100 x 100

Result – Real Dataset

DS-Softmax achieves **significant speedup** in three tasks and four dataset **without loss of performance** for theorem and real device

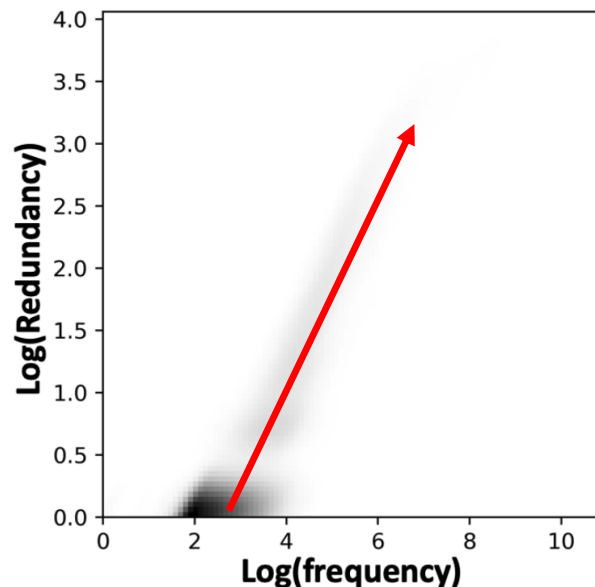
- Number of classes: 10000, 33278, 7709, 3740
- Even boost language modelling performance
- In Wiki-2, number of words = 33,278
 - **23x** Theoretical Reduction
 - **20x** Real Device Reduction

Task	Full		SVD-10			D-Softmax			DS-64 (Ours)		
	Value	ms	Value	FLOPs	ms	Value	FLOPs	ms	Value	FLOPs	ms
PTB	0.252	0.73	0.251	5.00×	0.18	0.245	2.00×	0.36	0.258	15.99×	0.05
Wiki-2	0.257	3.07	0.255	5.38×	0.60	0.256	2.00×	1.59	0.259	23.86×	0.15
En-Ve	25.2	1.91	25.1	5.06×	0.42	24.8	2.00×	0.98	25.0	15.08×	0.13
CASIA	90.6	1.61	90.2	2.61×	0.68	-	-	-	90.1	6.91×	0.25

Result – Interpretation

Higher frequency words appear in more experts.

- Similar in topic model¹
- High frequency words requires more expressive models²



The smallest expert in PTB, where 64 words left

- **Time is Money !!!**

Money

- million, billion, trillion, earnings, share, rate, stake, bond, cents, bid, cash, fine, payable

Time

- years, while, since, before, early, late, yesterday, annual, currently, monthly, annually, Monday, Tuesday, Wednesday, Thursday, Friday

Comparison

- up, down, under, above, below, next, though, against, during, within, including, range, higher, lower, drop, rise, growth, increase, less, compared, unchanged